2506/203
2507/203
ENGINEERING MATHEMATICS II
June/July 2020
Time: 3 hours

2506/203
Airframe and Engines

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING

(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non-programmable scientific calculator.

*Answer any **FIVE** of the following **EIGHT** questions.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given the vectors $\underset{\sim}{A} = -2\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{B} = 3\underset{\sim}{i} - \underset{\sim}{j} + 5\underset{\sim}{k}$, determine:

(i) a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(9 marks)

(b) A scalar field is given by $\phi = x^2z^2 + 2y^3$. Determine, at the point (1, -1, 2):

(i) $|\nabla\phi|$

ϕ-directional
$\nabla\phi$- Gradient
$\nabla\times\phi$-curl

(ii) the directional derivative of ϕ in the direction of the vector $\underset{\sim}{A} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

(7 marks)

(c) Determine the divergence of the electric field vector

$\underset{\sim}{E} = (x^2 + 2y^2)\underset{\sim}{i} + (y^2 + z)\underset{\sim}{j} + (x^2 - z^2)\underset{\sim}{k}$ at the point (1, 1, –1). (4 marks)

2. (a) Given the matrices $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$,

(i) determine a matrix C = 4A - 2B;

(ii) show that $(BA)^T = A^T B^T$.

(10 marks)

(b) Three currents, I_1, I_2 and I_3 in amperes, in a d.c network satisfy the simultaneous equations:

$I_1 + 2I_2 - I_3 = 5$
$-I_1 + 3I_2 + 2I_3 = 12$
$2I_1 - I_2 + I_3 = 1$

determinant

Use the inverse matrix method to solve the equations. (10 marks)

3. (a) Show that the general solution of the differential equation $x\dfrac{dy}{dx} = y + x^2y$ may be expressed in the form $y = A\,x\,e^{\frac{1}{2}x^2}$, where A is an arbitrary constant. (8 marks)

(b) The charge $q(t)$ on the plates of a capacitor satisfies the differential equation $\dfrac{d^2q}{dt^2} + 3\dfrac{dq}{dt} + 2q = \sin t$. Use the D-operator method to find an expression for $q(t)$, given that when $t = 0, q = 0$ and $\dfrac{dq}{dt} = 0$. (12 marks)

4. (a) Given $z = x^3 \cos\left(\frac{y}{x}\right)$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$. (5 marks)

(b) The power dissipated in a resistor of resistance R Ohms is given by $P = \frac{V^2}{R}$ Watts, where V is the voltage across the resistor in volts. Use partial differentiation to determine the percentage charge in P if V increases by 2% and R increases by 3%. (5 marks)

(c) Locate the stationery points of the function $z = x^3 + 6xy + 3y^2 + 6$, and determine their nature. (10 marks)

5. (a) (i) Use Maclaurins theorem to expand $\sin\left(x + \frac{\pi}{6}\right)$ as far as the fourth term.

(ii) Determine the approximate value of sin $33\frac{1}{2}°$ correct to four decimal places, using the result in (i).

(10 marks)

(b) (i) Expand $x^3 + 2x^2 - 3x + 1$ in a Taylor series about the point $x = 1$.

(ii) Hence, evaluate the integral

$$\int_2^3 \frac{x^3 + 2x^2 - 3x + 1}{(x-1)^2}\,dx.$$

(10 marks)

6. (a) Find the:

(i) Laplace transform of $f(t) = t\cos 6t$;

(ii) inverse Laplace transform of $F(s) = \frac{s+1}{(s-1)(s^2+5)}$.

(9 marks)

(b) The circuit in figure 1 is dead prior to switch closure at $t = 0$.

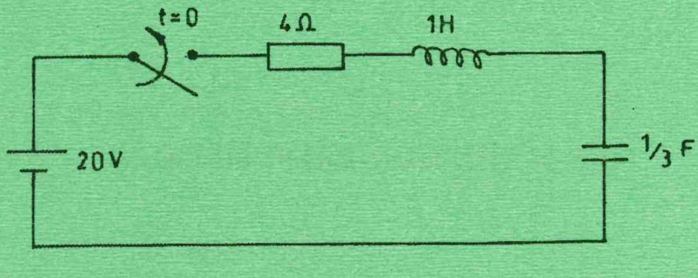

Fig.1

Use Laplace transforms to determine expressions for the:

(i) charge $q(t)$ on the capacitor;

(ii) current $i(t)$ in the circuit, for $t \geq 0$.

(11 marks)

7. (a) Show that the general solution of the differential equation $(x^2+y^2)\frac{dy}{dx}=2xy$ may be expressed in the form $x^2+y^2=cy$, where c is an arbitrary constant. (9 marks)

(b) Use the method of undetermined coefficients to solve the differential equation $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+4x=e^{-2t}$, given that when $t=0, x=0$ and $\frac{dx}{dt}=0$. (11 marks)

8. (a) Table I shows marks scored by 113 students in a mathematics test.

Table I

Marks scored	0 - 6	7 - 12	13 - 18	19 - 24	25 - 30	31 - 36	37 - 42
Frequency	6	11	25	35	18	12	6

Determine the:

(i) mean;

(ii) mode;

(iii) median.

(9 marks)

(b) A continuous random variable x has a probability density function $f(x)$ defined by

$$f(x)=\begin{cases} cx(10-x), & 0<x<1 \\ 0 \quad , & elsewhere \end{cases}$$

Determine the:

(i) value of the constant c;

(ii) mean of x;

(iii) variance of x.

(11 marks)

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