

2101/301      2106/301  
2102/301      2107/301  
2103/301      2108/301  
2104/301  
2105/301  
**MATHEMATICS**  
Oct./Nov. 2004  
Time: 3 hours

**THE KENYA NATIONAL EXAMINATIONS COUNCIL**

**DIPLOMA IN MECHANICAL ENGINEERING  
(PRODUCTION OPTION)**

**DIPLOMA IN MECHANICAL ENGINEERING  
(PLANT OPTION)**

**DIPLOMA IN AUTOMOTIVE ENGINEERING**

**DIPLOMA IN CONSTRUCTION PLANT ENGINEERING**

**DIPLOMA IN AGRICULTURAL ENGINEERING  
(FARM POWER AND MACHINERY OPTION)**

**DIPLOMA IN MECHANICAL ENGINEERING  
(FABRICATION TECHNOLOGY AND METALLURGY OPTION)**

**DIPLOMA IN AERONAUTICAL ENGINEERING  
(AIRFRAMES AND ENGINES OPTION)**

**DIPLOMA IN MECHANICAL ENGINEERING  
(MATERIALS TECHNOLOGY AND METALLURGY OPTION)**

**MATHEMATICS**

3 hours

**INSTRUCTIONS TO CANDIDATES**

You should have the following for this examination:

Answer book

Mathematical tables/calculator.

Answer any **FIVE** of the following **EIGHT** questions.

All questions carry equal marks.

Maximum marks for each part of a question are indicated.

Abridged tables of Laplace transforms and the standard normal distributions are included.

**This paper consists of 4 printed pages**

1. (a) Given matrix  $M = \begin{pmatrix} 3 & -6 & 2 \\ 6 & 2 & -3 \\ 2 & 3 & 6 \end{pmatrix}$  find  $MM^T$  and hence  $M^{-1}$ .

(2 marks)

(b) Find the inverse of the matrix

$$P = \begin{pmatrix} 4 & 8 & 3 \\ 3 & 5 & 2 \\ 2 & 4 & 3 \end{pmatrix}$$

(8 marks)

(c) Use Cramer's rule to solve the equations

$$3p - q + 2r = 3$$

$$2p + 3q - r = -1$$

$$p + 4q - 5r = 8.$$

(10 marks)

2. (a) Find the inverse Laplace transform of

$$\frac{2s+5}{s^2+4s+7}$$

(3 marks)

(b) Derive the Laplace transform of  $t \cosh 3t$  from the first principles.

(8 marks)

(c) Use Laplace transforms to solve the differential equation,

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = e^{3t} \text{ given that } t=0, y=3, \frac{dy}{dt} = 4.$$

(9 marks)

3. (a) Given  $\Phi = x^2yz - 4y^2z - 3$  find the directional derivative of  $\Phi$ , at the point  $(1, -1, 2)$  in the direction  $\mathbf{r} = 5\mathbf{i} + 10\mathbf{j} - 10\mathbf{k}$ .

(8 marks)

(b) Find, the co-ordinates  $(x, y, z)$  of the stationary points of the function  $Z = x^3 + y^2 - 2xy - x + 5$  and determine their nature.

(12 marks)

4. (a) Use Newton Raphson's formula and three iterations to obtain the root of the equation  $6x^4 - 15x^2 - 1 = 0$  taking  $X_0 = 1.6$ . Give the answer correct to five decimal places.

(7 marks)

(b) Table 1 gives data obtained in an experiment carried out in a workshop.

Table 1

x	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3
f(x)	-1.143	-0.932	-0.901	-0.900	-0.899	-0.868	-0.657
x	0.4	0.5					
f(x)	0.124	2.225					

Workout a difference table of the data and using Gregory-Newton formulae evaluate correct to four significant figures;

(i)  $f(-0.18)$

(ii)  $f(0.42)$

(13 marks)

5. (a) Find an expression for  $r$  if  $\tan 2x \frac{dr}{dx} - r = \sin^2 2x$  given that  $x = \frac{\pi}{4}$ ,  $r = 4$ .

(10 marks)

(b) Find the general solution of the differential equation;

$$3 \frac{d^2x}{dt^2} + 2 \frac{dx}{dt} = 2t^2 - 1$$

(10 marks)

6. A function  $f(t)$  is defined by

$$f(t) = 0 - 2 \leq t < 0$$

$$= \left(3 - \frac{3}{2}t\right) 0 \leq t \leq 2$$

$$f(t) = f(t+4)$$

(i) Obtain the Fourier series for the function  $f(t)$  and

(ii) Calculate the percentage third harmonic

(20 marks)

7. (a) A machine produces components whose masses are normally distributed with mean  $\mu$  and standard deviation  $\sigma$ . If 89.8% of the components have a mass of at least 88g and 3% have a mass less than 84.5g; find the mean and the standard deviation of the distribution. (6 marks)

(b) The diameters of bolts produced by a certain machine are distributed by a probability density function

$$f(x) = \begin{cases} kx(3-x) & 1 < x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find the:

- (i) constant K
- (ii) probability that the diameter of a bolt selected at a random will fall in the interval  $1 < x < 2.5$
- (iii) mean and the variance of the distribution.

(14 marks)

8. (a) (i) Change the order of the double integral  $\int_0^6 \int_{\frac{x}{3}}^2 ye^{y^3} dy dx$  and hence evaluate the integral

(ii) Work out the triple integration

$$\int_1^2 \int_x^{3\sqrt{3}y} \int_0^y \frac{y}{y^2+z^2} dz dy dx \text{ and hence evaluate the integral}$$

(14 marks)

(b) Find the volume of the tetrahedron bounded by the planes  $X=0$ ,  $y=0$  and  $3x + 8y + 4z - 24 = 0$ .

(6 marks)