2506/203
2507/203
ENGINEERING MATHEMATICS II
June/July 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
A bridged table of Laplace transforms is attached.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrices $A=\begin{bmatrix}-1 & 1 & 2\\ 1 & -1 & 2\\ 0 & 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & -1\\ 1 & -1 & 1\\ 1 & -1 & 0\end{bmatrix}$, determine:

(i) the matrix $C=A^2-3B$;

(ii) A^{-1}.

(11 marks)

(b) Three currents I_1, I_2 and I_3 in amperes, flowing in a circuit satisfy the simultaneous equations

$$I_1-I_2+I_3=-1$$
$$I_1+2I_2-I_3=6$$
$$-2I_1-I_2+3I_3=4$$

Use Cramer's rule to solve the equations. (9 marks)

2. (a) Given $z=\tan^{-1}(x^2-y^2)$, show that $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=0$. (5 marks)

(b) Use partial differentiation to determine the equations of the:

(i) tangent;
(ii) normal;

to the curve $z=x^2-y^2+3xy-2x+y$ at the point (0, 0). (7 marks)

(c) Locate the stationary points of the function $z=x^3-3y^2+6xy$, and determine their nature.

(8 marks)

3. (a) Use Maclaurin's theorem to expand:

(i) $\cos x$, as far as the term in x^2;

(ii) $\frac{1}{1+x}$, as far as the term in x^3.

(iii) Hence, show that $\frac{\cos x}{1+x}=1-x+\frac{1}{2}x^2-\frac{1}{2}x^3$ approximately, and evaluate the integral $\int_0^1 \frac{\cos x}{1+x}dx$, giving the answer in fraction form. (13 marks)

(b) (i) Expand $\sin\left(\frac{\pi}{2}+h\right)$ in a Taylor's series as far as the term in h^3.

(ii) Use the result in (i) to determine the approximate value of $\sin 9\frac{1}{2}°$, correct to four decimal places.

(7 marks)

4. (a) Obtain the general solution of the differential equation:

$$x^3\frac{dy}{dx}+x^2(1-2x^4)y=x^3.$$

(8 marks)

(b) The charge q(t) on the plates of a capacitor satisfies the differential equation

$$\frac{d^2q}{dt^2}+2\frac{dq}{dt}+2q=e^{-t}.$$

Given that when $t=0$, $q=0$ and $\frac{dq}{dt}=0$, use the method of undetermined coefficients to solve the equation.

(12 marks)

5. (a) The three vectors $\underset{\sim}{A}=\underset{\sim}{i}+\underset{\sim}{j}+a\underset{\sim}{k}$, $\underset{\sim}{B}=2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$ and $\underset{\sim}{C}=\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$ are coplanar. Determine the value of the constant a.

(5 marks)

(b) Determine a unit vector that is perpendicular to the vectors $\underset{\sim}{A}=2\underset{\sim}{i}-\underset{\sim}{j}-2\underset{\sim}{k}$ and $\underset{\sim}{B}=-\underset{\sim}{i}-2\underset{\sim}{j}-2\underset{\sim}{k}$.

(7 marks)

(c) The electric field $\underset{\sim}{E}=xz\underset{\sim}{i}+yz\underset{\sim}{j}+z^2\underset{\sim}{k}$ exists in a region of space. Determine, the point $(1,-1,1)$:

(i) $|\nabla\times\underset{\sim}{E}|$

(ii) $\nabla\cdot\underset{\sim}{E}$

(8 marks)

6. (a) Find the:

(i) Laplace transform of $f(t)=t\cosh t$

(ii) inverse Laplace transform of $\frac{2s-1}{(s+1)(s^2+9)}$.

(9 marks)

(b) The network in Figure 1 is dead prior to switch closure at $t=0$.

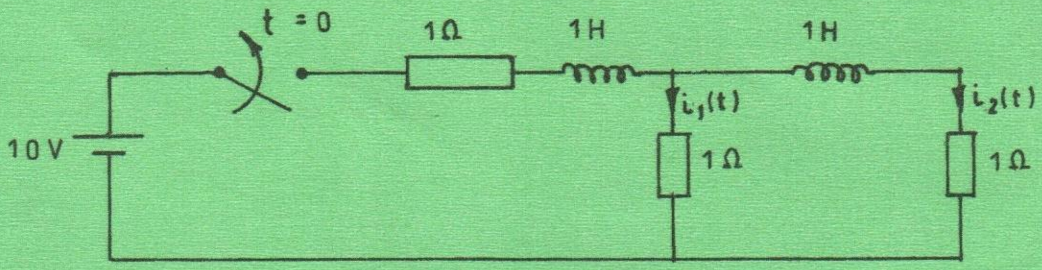


Fig. 1

Use Laplace transforms to find an expression for the current $i_2(t)$, for $t\geq 0$.

(11 marks)

7. (a) Obtain the general solution of the differential equation:

$$x^2 \frac{dy}{dx} = 2xy + x^2 + y^2 .$$

(8 marks)

(b) A dynamic system is modelled by the differential equation

$2\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = e^{-2t}$. Use the D-operator method to solve the equation,

given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 1$.

(12 marks)

8. A random variable x has a probability density function $f(x)$ given by:

$$f(x) = \begin{cases} k(3x^2 - x^3), & 0 < x < 3 \\ 0, & elsewhere \end{cases}.$$

Determine the:

(a) value of the constant k;

(b) mean and standard deviation;

(c) mode;

(d) $P(1 < x < 2)$.

(20 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\,t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

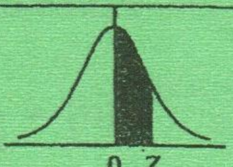
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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