

2506/103

2507/103

ENGINEERING MATHEMATICS I  
AND ENGINEERING SCIENCE I

June/July 2019

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING  
(AIRFRAMES AND ENGINES OPTION)  
(AVIONICS OPTION)

MODULE I

ENGINEERING MATHEMATICS I AND ENGINEERING SCIENCE I

3 hours

### INSTRUCTIONS TO CANDIDATES

*You should have the following for this examination:*

*Answer booklet;*

*Drawing instruments;*

*Mathematical tables/Non-programmable scientific calculator.*

*This paper consists of EIGHT questions in TWO sections; A and B.*

*Answer any THREE questions from section A and any TWO questions from section B.*

*All questions carry equal marks.*

*Maximum marks for each part of a question are as indicated.*

*Candidates should answer the questions in English.*

**This paper consists of 5 printed pages.**

**Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**

## SECTION A: ENGINEERING MATHEMATICS

Answer any **THREE** questions from this section.

1. (a) Solve the equation  $Z^3 = j$ , leaving the answer in the form  $a + jb$ . (8 marks)
- (b) (i) Express  $\operatorname{sech}^{-1} x$  in logarithmic form;
- (ii) hence determine the value of  $\operatorname{sech}^{-1} 0.25$ , correct to three decimal places. (8 marks)
- (c) Solve the equation:
- $$6x^2 - x - 2 = 0. \quad (4 \text{ marks})$$
2. (a) Solve the equations:
- (i)  $5^{2x+1} = 8^{x+1}$ ;
- (ii)  $\log_2 x + \log_8 x = 8$ ;
- correct to three significant figures. (10 marks)
- (b) Three currents  $I_1, I_2$  and  $I_3$  in amperes in an electrical circuit satisfy the equations:
- $$4I_1 - I_2 + I_3 = 12$$
- $$3I_1 + 2I_2 - 2I_3 = -2$$
- $$5I_1 + 3I_2 - I_3 = 0$$
- use elimination to solve the equations. (10 marks)
3. (a) Convert the equations:
- (i)  $(x-1)^2 + y^2 = 4$  to polar form;
- (ii)  $\frac{r^2}{16} - \frac{7}{144} r^2 \sin^2 \theta = 1$  to cartesian form. (7 marks)
- (b) Determine the values of the constants  $A$  and  $B$  in the equation:
- $$A \cosh x + B \sinh x = 5e^x - 7e^{-x}. \quad (6 \text{ marks})$$
- (c) (i) Use the binomial theorem to expand  $\left(\frac{1+3x}{1-3x}\right)^{\frac{1}{2}}$  up to the term in  $x^2$ ;
- (ii) hence evaluate  $\sqrt{\frac{13}{7}}$ . (7 marks)

4. (a) Eight men and four women are invited for an interview. If the vacancies are three, determine the number of ways that the vacancies can be filled if there has to be at least one person from either gender. (6 marks)

(b) Prove the identities:

(i)  $\frac{\sin^2 \theta}{1 + \cos \theta} = 1 - \cos \theta;$

(ii)  $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta.$

(7 marks)

(c) Solve the equation:

$6 \cos^2 \theta - \sin \theta - 5 = 0$  for  $0^\circ \leq \theta \leq 360^\circ$  inclusive.

(7 marks)

5. (a) Find  $\frac{dy}{dx}$  for the functions:

(i)  $y = \frac{\cos x + \sin x}{x^2};$

(ii)  $y = \ln(x^2 + 7).$

(6 marks)

(b) Use implicit differentiation to determine the equation of the tangent to the curve:

$y^2 + 4xy + x - 13 = 0$  at the point (1, 2).

(6 marks)

(c) Evaluate the integrals:

(i)  $\int_0^{\frac{\pi}{2}} x \cos x \, dx;$

(ii)  $\int_0^1 \frac{5x + 7}{(x + 1)(x + 2)} \, dx.$

(8 marks)

## SECTION B: ENGINEERING SCIENCE

Answer any **TWO** questions from this section.

6. (a) Two forces  $F_1 = 20N$  and  $F_2 = 30N$  act at a point. If the angle between the two forces is  $30^\circ$ , calculate the:
- magnitude of the resultant force;
  - direction of the resultant force. (6 marks)
- (b) A block of mass 40 kg slides down a plane which is inclined at  $10^\circ$  to the horizontal. If the co-efficient of friction between the block and the plane is 0.1, calculate the:
- frictional force;
  - net force experienced by the block in the direction parallel to the plane.
- (Take:  $g = 9.8 m/s^2$ ). (6 marks)
- (c) (i) Define the terms:
- work;
  - potential energy;
  - kinetic energy.
- (ii) An aeroplane of mass 20,000 kg runs along a level runway at a constant speed of 50 m/s. If it experiences a total retarding force of 200 kN, calculate the:
- power developed by the engine;
  - kinetic energy of the plane. (8 marks)
7. (a) State the **three** modes of heat transfer. (3 marks)
- (b) A wooden block of volume  $0.125 m^3$  and density  $0.7 g/cm^3$  floats in a liquid with  $\frac{2}{3}$  of its volume submerged. Determine the downward force which must be exerted on the block to fully immerse it in the liquid. (6 marks)
- (c) Air is compressed in an engine cylinder. The ratio of the initial to final volume is 8:1. If the initial temperature, initial pressure and final pressure are  $27^\circ C$ ,  $1 \times 10^5 Pa$  and  $20 \times 10^5 Pa$  respectively, calculate the final temperature. (5 marks)
- (d) A load of 27,000 N is raised through 60 cm by an effort of 3,600 N when the effort moves through 540 cm. Determine the efficiency of the machine at this load. (6 marks)

8. (a) Balance the chemical equation:



(b) Outline the **three** types of hydrocarbons. (3 marks)

(c) Given that the mass numbers of carbon and hydrogen are 12 and 1 respectively, determine the molecular mass of the chemical compound given by the formula:



(d) Calculate the mercury pressure at a depth of 20 cm if its density is  $13.6 \text{ g/cm}^3$  at a location where  $g = 9.8 \text{ m/s}^2$ . (3 marks)

(e) An electromagnetic wave of frequency  $4 \times 10^{16} \text{ Hz}$  travels in glass at a speed of  $2 \times 10^8 \text{ m/s}$ . Calculate its wavelength. (3 marks)

(f) Figure 1 shows a 5 m long beam of uniform cross-section supported at both ends. It carries a load of 4,000 N as shown. Determine the reaction forces at the supports. (4 marks)

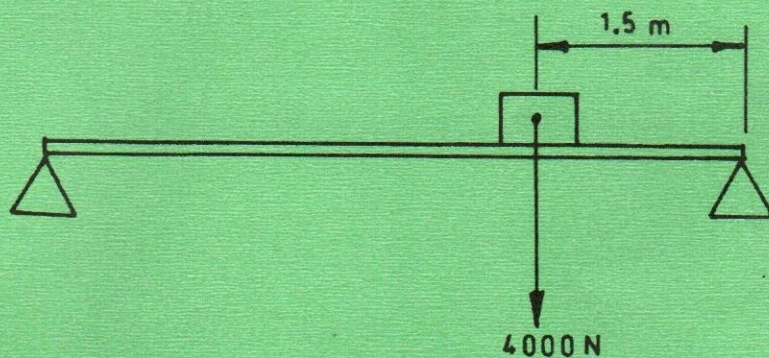


Fig. 1

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