

**2101-8/301**

**MATHEMATICS**

**Oct./Nov. 2010**

**Time: 3 hours**

**THE KENYA NATIONAL EXAMINATIONS COUNCIL**

**DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION)**

**DIPLOMA IN MECHANICAL ENGINEERING (PLANT OPTION)**

**DIPLOMA IN AUTOMOTIVE ENGINEERING**

**DIPLOMA IN CONSTRUCTION PLANT ENGINEERING**

**DIPLOMA IN AGRICULTURAL ENGINEERING**

**(FARM POWER MACHINERY OPTION)**

**DIPLOMA IN MECHANICAL ENGINEERING**

**(FABRICATION TECHNOLOGY AND METALLURGY OPTION)**

**DIPLOMA IN AERONAUTICAL ENGINEERING**

**DIPLOMA IN MECHANICAL ENGINEERING**

**(MATERIALS TECHNOLOGY AND METALLURGY OPTION)**

**MATHEMATICS**

**3 hours**

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination:*

*Answer booklet;*

*Mathematical tables/Scientific calculator.*

**Answer any FIVE of the EIGHT questions in this paper.**

**All questions carry equal marks.**

**Maximum marks for each part of a question are indicated.**

**Table of Laplace transforms and the standard normal distribution are included.**

**This paper consists of 8 printed pages.**

**Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**

1. (a) Given that  $A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \\ 3 & -1 & -1 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & -1 & 1 \\ 4 & -10 & -8 \\ -1 & 7 & 5 \end{pmatrix}$

(i) Find the matrix product AB

(ii) Hence solve the following system of simultaneous equations:

$$x + 2y + 3z = -6$$

$$-2x + y + 2z = 1$$

$$3x - y - z = 1$$

(8 marks)

- (b) A company makes three types of security locks A, B and C, each of which requires cutting, assembly, and finishing. Each unit of A requires 2 hours for cutting, 1 hour for assembly, and 3 hours for finishing. Each unit of B requires 1 hour for cutting, 2 hours for assembly, and 1 hour for finishing. Each unit of C requires 1 hour for cutting, 2 hours for assembly, and 3 hours for finishing.

Available machine resources provide exactly 10 hours for cutting, 14 hours for assembly, and 18 hours for finishing each week.

(i) Use the information provided to form a system of simultaneous linear equations.

(ii) Apply Cramer's rule to calculate the number of locks of each type produced given that there is optimum utilization of machine time at the factory in a week.

(12 marks)

2. (a) Given that  $u = e^x [\sin(y+z) - y \cos(y+z)]$

$$\text{Show that } \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + u = 2e^x \sin(y+z)$$

(8 marks)

- (b) Given the function  $f(x, y) = 5x^2 + 2xy + y^2 + y^2 + 8y - 16x - 4$ , find the stationary points and their nature.

(7 marks)

$$\begin{matrix} A & (2 & 1 & 3) \\ B & (1 & 2 & 1) \\ C & (1 & 2 & 3) \end{matrix} = \begin{pmatrix} 10 \\ 14 \\ 18 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \\ 18 \end{pmatrix}$$

- (c) Concrete at a mining site is poured onto a conical pile. At a certain time, the height is 28 metres and increasing at 15cm per minute and the radius is 12 metres and increasing at 1.9cm per minute. How fast is the volume increasing at that instant? (5 marks)

3. (a) Solve the differential equation.

$$xy \frac{dy}{dx} = x^2 + 2y^2 + 2xy$$

given that when  $x = 1, y = 0$

(8 marks)

- (b) A particle moves along a straight line such that its displacement  $X$  from a fixed point P is given by:

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = \cos 2t.$$

Find its displacement  $x$  at any time  $t$ , given that the particle starts from rest.

(12 marks)

4. (a) Find the Fourier series for the function.

$$f(x) = \begin{cases} 0 & \text{for } -\pi < x \leq -\frac{\pi}{3} \\ 2 & \text{for } -\frac{\pi}{3} < x \leq \frac{\pi}{3} \\ 1 & \text{for } \frac{\pi}{3} < x \leq \pi \end{cases}$$

$$f(x + 2\pi) = f(x) \quad (11 \text{ marks})$$

- (b) Find the half-range Fourier sine series for the function.

$$f(t) = \begin{cases} 4t & \text{for } 0 < t \leq 1 \\ -2t + 6 & \text{for } 1 < t \leq 3 \end{cases}$$

$$f(t+6) = f(t) \quad (9 \text{ marks})$$

5. (a) Evaluate the double integral.

$$\int_0^{3/2} \int_0^{\sqrt{9-y^2}} (x^2 + y^2) dx dy \quad (8 \text{ marks})$$

- (b) Find the volume V of the solid bounded on top by the surface  $z = 5 - x^2 - y$ , below by the  $x - y$  plane and the cylinder  $x^2 + y^2 = 16$ . (12 marks)

6.

- (a) Six points on the graph of the function  $y = P(x)$  are given by the  $(x, y)$  pairs:

$$(0.0, 1.2840), (0.2, 1.3499), (0.4, 1.4191), (0.6, 1.4918), (0.8, 1.5683), (1.0, 1.6490)$$

Use the Newton-Gregory forward differences interpolation formula to compute  $P(0.28)$  and  $P(-0.05)$  correct to four significant figures. (8 marks)



- (b) Show that:

(i) the equation  $x^3 - 3x^2 - 4 = 0$  has a root between 3 and 4;

(ii) using the Newton-Raphson method, the expression.

$$x_{n+1} = \frac{2x_n^3 - 3x_n^2 + 4}{3x_n^2 - 6x_n}$$
 can be used to approximate this root.

Determine the value of this root correct to four significant figures, taking  $x_0 = 3.5$ .

(12 marks)

7.

- (a) Find the laplace transform of  $f(t) = \sin at \cos ht$  at. (3 marks)

- (b) Find the inverse laplace transform of  $\frac{s^2 + 7}{s^4 - 1}$  (8 marks)



- (c) Use laplace transforms to solve the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 4 \text{ given that at } t = 0, y = 0 \text{ and } \frac{dy}{dt} = -1.$$

(9 marks)

8\*

(a)

A binomially distributed random variable  $X$  has expected value,  $E(x) = 27.52$  and variance,  $\text{Var}(x) = 15.69$ . Find the value of the two parameters  $n$  and  $p$ , where  $n$  is the sample size and  $P$  is the probability of success. (4 marks)

- (b) The number of non-productive hours in a factory can be modelled by the continuous random variable  $X$  with p.d.f  $f(x)$  given by

$$f(x) = \begin{cases} K(4x - x^2) & \text{for } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find:

- (i) the value of constant  $K$ ;
- (ii) the expected value  $E(x)$ ;
- (iii) the mode of  $x$ ;
- (iv)  $P(x \geq z^2)$ .

(8 marks)

(c)

A plant produces steel rods whose weights are known to be normally distributed with a standard deviation of 2.5kg. A random sample of 40 rods had a mean weight of 33kg.

Find:

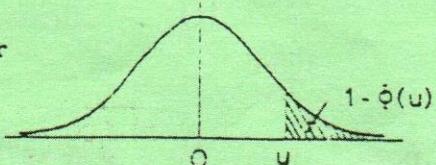
- (i) the 99% confidence limits for the population mean.
- (ii) the probability that a steel rod had weight of 35kg or more. (8 marks)

Table 3

## AREAS IN TAIL OF THE NORMAL DISTRIBUTION

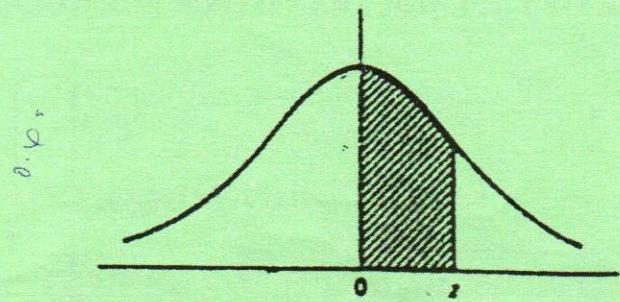
The function tabulated is  $1 - \Phi(u)$  where  $\Phi(u)$  is the cumulative distribution function of a standardised Normal variable  $u$ . Thus  $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-u^2/2} du$  is the probability that a

standardised Normal variable selected at random will be greater than a value of  $u$  ( $= \frac{x-\mu}{\sigma}$ )



| $\frac{(x - \mu)}{\sigma}$ | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
|----------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0                        | .5000  | .4960  | .4920  | .4880  | .4840  | .4801  | .4761  | .4721  | .4681  | .4641  |
| 0.1                        | .4602  | .4562  | .4522  | .4483  | .4443  | .4404  | .4364  | .4325  | .4286  | .4247  |
| 0.2                        | .4207  | .4168  | .4129  | .4090  | .4052  | .4013  | .3974  | .3936  | .3897  | .3859  |
| 0.3                        | .3821  | .3783  | .3745  | .3707  | .3669  | .3632  | .3594  | .3557  | .3520  | .3483  |
| 0.4                        | .3446  | .3409  | .3372  | .3336  | .3300  | .3264  | .3228  | .3192  | .3156  | .3121  |
| 0.5                        | .3085  | .3050  | .3015  | .2981  | .2946  | .2912  | .2877  | .2843  | .2810  | .2776  |
| 0.6                        | .2743  | .2709  | .2676  | .2643  | .2611  | .2578  | .2546  | .2514  | .2483  | .2451  |
| 0.7                        | .2420  | .2389  | .2358  | .2327  | .2296  | .2266  | .2236  | .2206  | .2177  | .2148  |
| 0.8                        | .2119  | .2090  | .2061  | .2033  | .2005  | .1977  | .1949  | .1922  | .1894  | .1867  |
| 0.9                        | .1841  | .1814  | .1788  | .1762  | .1736  | .1711  | .1685  | .1660  | .1635  | .1611  |
| 1.0                        | .1587  | .1562  | .1539  | .1515  | .1492  | .1469  | .1446  | .1423  | .1401  | .1379  |
| 1.1                        | .1357  | .1335  | .1314  | .1292  | .1271  | .1251  | .1230  | .1210  | .1190  | .1170  |
| 1.2                        | .1151  | .1131  | .1112  | .1093  | .1075  | .1056  | .1038  | .1020  | .1003  | .0985  |
| 1.3                        | .0968  | .0951  | .0934  | .0918  | .0901  | .0885  | .0869  | .0853  | .0838  | .0823  |
| 1.4                        | .0808  | .0793  | .0778  | .0764  | .0749  | .0735  | .0721  | .0708  | .0694  | .0681  |
| 1.5                        | .0668  | .0655  | .0643  | .0630  | .0618  | .0606  | .0594  | .0582  | .0571  | .0559  |
| 1.6                        | .0548  | .0537  | .0526  | .0516  | .0505  | .0495  | .0485  | .0475  | .0465  | .0455  |
| 1.7                        | .0446  | .0436  | .0427  | .0418  | .0409  | .0401  | .0392  | .0384  | .0375  | .0367  |
| 1.8                        | .0359  | .0351  | .0344  | .0336  | .0329  | .0322  | .0314  | .0307  | .0301  | .0294  |
| 1.9                        | .0287  | .0281  | .0274  | .0268  | .0262  | .0256  | .0250  | .0244  | .0239  | .0233  |
| 2.0                        | .02275 | .02222 | .02169 | .02118 | .02068 | .02018 | .01970 | .01923 | .01876 | .01831 |
| 2.1                        | .01786 | .01743 | .01700 | .01659 | .01618 | .01578 | .01539 | .01500 | .01463 | .01426 |
| 2.2                        | .01390 | .01355 | .01321 | .01287 | .01255 | .01222 | .01191 | .01160 | .01130 | .01101 |
| 2.3                        | .01072 | .01044 | .01017 | .00990 | .00964 | .00939 | .00914 | .00889 | .00866 | .00842 |
| 2.4                        | .00820 | .00798 | .00776 | .00755 | .00734 | .00714 | .00695 | .00676 | .00657 | .00639 |
| 2.5                        | .00621 | .00604 | .00587 | .00570 | .00554 | .00539 | .00523 | .00508 | .00494 | .00480 |
| 2.6                        | .00466 | .00453 | .00440 | .00427 | .00415 | .00402 | .00391 | .00379 | .00368 | .00357 |
| 2.7                        | .00347 | .00336 | .00326 | .00317 | .00307 | .00298 | .00289 | .00280 | .00272 | .00264 |
| 2.8                        | .00256 | .00248 | .00240 | .00233 | .00226 | .00219 | .00212 | .00205 | .00199 | .00193 |
| 2.9                        | .00187 | .00181 | .00175 | .00169 | .00164 | .00159 | .00154 | .00149 | .00144 | .00139 |
| 3.0                        | .00135 |        |        |        |        |        |        |        |        |        |
| 3.1                        | .00097 |        |        |        |        |        |        |        |        |        |
| 3.2                        | .00069 |        |        |        |        |        |        |        |        |        |
| 3.3                        | .00048 |        |        |        |        |        |        |        |        |        |
| 3.4                        | .00034 |        |        |        |        |        |        |        |        |        |
| 3.5                        | .00023 |        |        |        |        |        |        |        |        |        |
| 3.6                        | .00016 |        |        |        |        |        |        |        |        |        |
| 3.7                        | .00011 |        |        |        |        |        |        |        |        |        |
| 3.8                        | .00007 |        |        |        |        |        |        |        |        |        |
| 3.9                        | .00005 |        |        |        |        |        |        |        |        |        |
| 4.0                        | .00003 |        |        |        |        |        |        |        |        |        |

Partial areas under the  
standardised normal curve



| $z = \frac{x - \bar{x}}{\sigma}$ | 0      | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      |
|----------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0                              | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0159 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1                              | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0673 | 0.0714 | 0.0753 |
| 0.2                              | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3                              | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4                              | 0.1554 | 0.1891 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5                              | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2086 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6                              | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7                              | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2760 | 0.2794 | 0.2823 | 0.2852 |
| 0.8                              | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9                              | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3215 | 0.3340 | 0.3365 | 0.3389 |
| 1.0                              | 0.3413 | 0.3438 | 0.3451 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1                              | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2                              | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3                              | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4                              | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5                              | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4430 | 0.4441 |
| 1.6                              | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7                              | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8                              | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9                              | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4762 | 0.4767 |
| 2.0                              | 0.4772 | 0.4778 | 0.4783 | 0.4785 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1                              | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2                              | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4882 | 0.4890 |
| 2.3                              | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4                              | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5                              | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6                              | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7                              | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8                              | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4980 | 0.4980 | 0.4981 |
| 2.9                              | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0                              | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1                              | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2                              | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3                              | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4                              | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5                              | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6                              | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7                              | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8                              | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9                              | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |

# TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!} t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2 + a^2}\right] = \frac{1}{a} \sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2 + a^2}\right] = \cos at$$

## First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n \mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u) du\right] = \frac{1}{s} \mathcal{L}[f(t)] \quad \mathcal{L}^{-1}\left[\frac{1}{s} F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)] du$$

In the following formulas,  $F(s) = \mathcal{L}[f(t)]$  so  $f(t) = \mathcal{L}^{-1}[F(s)]$ .

## First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a)$$

$$\mathcal{L}^{-1}[F(s)] = e^{at} \mathcal{L}^{-1}[F(s+a)]$$

## Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \mathcal{L}[f(t)]$$

$$\mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

## Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as} \mathcal{L}[g(t+a)] \quad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$