2101/301   2106/301
2102/301   2107/301
2103/301   2108/301
2104/301
2105/301
MATHEMATICS
Oct./Nov. 2009
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING
(PRODUCTION OPTION)
DIPLOMA IN MECHANICAL ENGINEERING
(PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING
DIPLOMA IN AGRICULTURAL ENGINEERING
(FARM POWER AND MACHINERY OPTION)
DIPLOMA IN MECHANICAL ENGINEERING
(FABRICATION TECHNOLOGY AND METALLURGY OPTION)
DIPLOMA IN AERONAUTICAL ENGINEERING
(AIR FRAME AND ENGINES OPTION)
DIPLOMA IN MECHANICAL ENGINEERING
(MATERIALS TECHNOLOGY AND METALLURGY OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
    Answer booklet
    Mathematical tables/scientific calculator.
Answer any FIVE of the EIGHT questions in this paper.
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Abridged tables of Laplace transforms and the standard normal distribution are included.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that
all the pages are printed as indicated and that no questions are missing.

    Turn over

1. (a) (i) Use Maclaurin's theorem to expand sin 3x upto the term in x^7.

   (ii) Hence, evaluate $\int_0^1 \dfrac{\sin 3x}{x^{1/4}} \, dx$ correct to 4 places of decimals. (8 marks)

   (b) Evaluate the triple integral

   $$\int_0^1 \int_{-1}^{2x} \int_0^{y+1} z^2 \, dz \, dy \, dx$$

   (6 marks)

   (c) Use a double integral to find the volume of the solid bounded by the surface $z = 4 - x^2 - y^2$ and the planes $x = 0$, $x = 1$, $y = 0$ and $y = -x+1$.

   (6 marks)

2. (a) The temperature θ (degrees) of a steel bar immersed in an atmosphere of varying temperature is given by

   $$10 \frac{d\theta}{dt} + \theta = 50 - 25t$$

   Find the temperature at time t if $\theta = 60°$ when $t = 0$. (8 marks)

   (b) The motion of a mass vibrating in a typical one-dimensional damping mechanism is given by the equation,

   $$\frac{d^2 x}{dt^2} - 2\frac{dx}{dt} + 2x = 5 \sin 3t$$

   Find x in terms of t. (12 marks)

3. (a) (i) Determine the unit vector $\vec{V}$, normal to the surface $6xy^3 - y^2z^2 + xyz - 4 = 0$ at the point $(1, -1, 4)$.
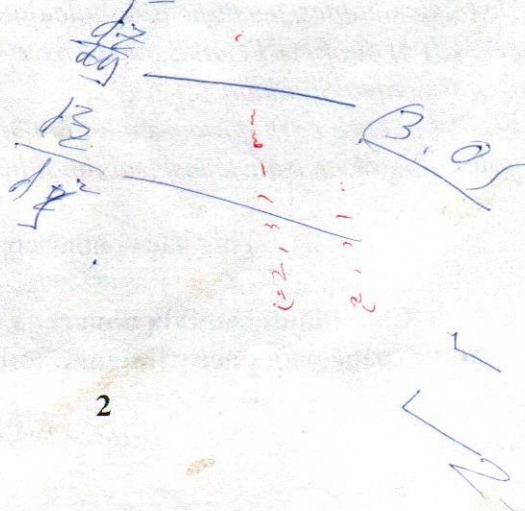
   (ii) Hence, determine the value of $\vec{V} \cdot \vec{A}$ if $\vec{A} = 1.8\underline{i} + 4\underline{j} + 5.4\underline{k}$ correct to 3 decimal places. (8 marks)

   (b) Find the stationary points of the function $z = 3y^3 - 2xy + 5y^2 - 4x^2$ and determine their nature. (12 marks)

(a) When a number of mass/spring systems are connected together, the following equation involving determinants is produced:

$$\begin{vmatrix} 1-x & -\frac{1}{2} & 0 \\ -\frac{3}{4} & \frac{3}{2}-x & -\frac{3}{4} \\ 0 & -\frac{3}{4} & 1-x \end{vmatrix} = 0$$

Show that x = 1 satisfies the equation and hence find the other values of x.

(7 marks)

(b) A factory is to install three types of machines A, B and C, each of which requires supervisors, operators and output managers. Type A machine needs 3 supervisors, 4 operators and 1 output manager. Type B needs 2 supervisors, 5 operators and 3 output managers. Type C needs 1 supervisor, 1 operator and 2 output managers. The factory requires 23 supervisors, 40 operators and 31 output managers. Using inverse matrix method, find how many machines of each type the factory requires.

(13 marks)

A function f (t) is defined by:

$$f(t) = \begin{cases} 2-t & -\pi < t < 0 \\ 2+t & 0 < t < \pi \\ f(t) = f(t+2\pi) \end{cases}$$

(a) Sketch the function f(t) using more than **two** periods.

(3 marks)

(b) Obtain the Fourier series for the function and hence deduce the series for $\frac{\pi^2}{8}$.

(17 marks)

(a) Derive the Laplace transform of $e^{-2t}\cos 3t$ using first principles.

(5 marks)

(b) Find $L^{-1}\left[\frac{2S^2 - 4S + 10}{(S^2 + 3)(S^2 - 1)}\right]$.

(7 marks)

(c) Use Laplace transforms to solve the equation.

$$\frac{2d^2y}{dt^2} - \frac{5dy}{dt} + 3y = e^t$$

given that when t = 0, y = 1 and $\frac{dy}{dt} = 2$.

(8 marks)

7. (a) The distribution of resistance in a metal shaft is known to be normally distributed with 10% shafts having a resistance exceeding 10.256 units and 5% having resistance smaller than 9.671 units. Find what percentage lies between 8.04 units and 9.55 units.

(6 marks)

(b) The vibratory stress on a turbine blade at any particular time follows a p.d.f. described as follows:

$$f_x = \begin{cases} \dfrac{x}{\theta^2} e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Verify that f(x) is a p.d.f.

(4 marks)

(c) Using the result in 7(b) above, and given that $\theta = 100$, find the:

(i) probability that x is at most 200;
(ii) median;
(iii) mode of the distribution.

(10 marks)

8. (a) The motion of a particle in an electrostatic field is given by the function $f(x) = 5x^3 - 2x^2 + x - 6$.

(i) Using Newton-Raphson method, show that a better approximation to the equation f(x) = 0 is give by:

$$x_{n+1} = \frac{10x_n^3 - 2x_n^2 + 6}{15x_n^2 - 4x_n + 1}$$

Hence find the root of the equation near $x_0 = 1$ correct to five decimal places.

(11 marks)

(b) The data below describe the path followed by the arm of a crane:

x	-1	0	1	2	3	4	5	6
f(x)	1	1	9	43	121	261	481	799

Using the Gregory-Newton backward difference formula, find the value of f(4.8).

(9 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad\qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!} t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad\qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2} \qquad\qquad \mathcal{L}^{-1}\left[\frac{1}{s^2 + a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2 + a^2} \qquad\qquad \mathcal{L}^{-1}\left[\frac{s}{s^2 + a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad\qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad\qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

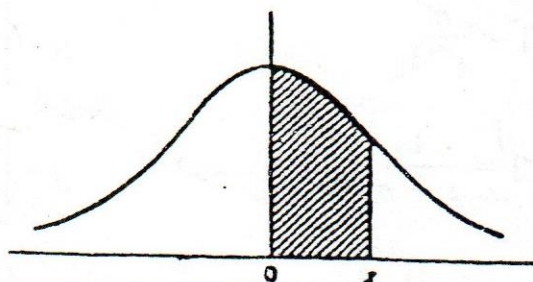$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad\qquad \mathcal{L}^{-1}\left[\frac{d^n F(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad\qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Partial areas under the
standardised normal curve

$z = \dfrac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000