2101/301 2105/301
2102/301 2106/301
2103/301 2107/301
2104/301 2108/301
MATHEMATICS
Oct./Nov. 2008
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION)
DIPLOMA IN MECHANICAL ENGINEERING (PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING
DIPLOMA IN AGRICULTURAL ENGINEERING (FARM POWER AND MACHINERY OPTION)
DIPLOMA IN MECHANICAL ENGINEERING (FABRICATION TECHNOLOGY AND METALLURGY OPTION)
DIPLOMA IN AERONAUTICAL ENGINEERING
DIPLOMA IN MECHANICAL ENGINEERING (MATERIALS TECHNOLOGY AND METALLURGY OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;
Mathematical Tables/Scientific Calculator

*Answer any **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Tables of Laplace transforms are included.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and no questions are missing.

1. (a) Use Maclaurin's theorem to expand:

$e^{2x} \sin 2x$ in ascending powers of x as far as the term in x^5. (9 marks)

(b) Show that the equation

$$x^3 - 4x^2 + 4 = 0$$

has a root between -1 and 0. Hence, taking $x_o = -0.5$, find the root correct to three decimal places, using Newton-Raphson method. (11 marks)

2. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 6 & 3 \\ 9 & -8 & -5 \end{bmatrix}$$

(7 marks)

(b) A manufacturer produces an alloy that is made from steel, aluminium and copper. The cost of 2 tonnes of steel, 1 ton of aluminium and 3 tonnes of copper is shs 190,000, the cost of 1 ton of steel, 5 tonnes of aluminium and 4 tonnes of copper is shs 260,000 while the cost of 6 tonnes of steel, 9 tonnes of aluminium and 3 tonnes of copper is shs 510,000. Determine the cost of each type of metal per ton using Cramer's rule.

(13 marks)

3. A function f(x) is defined by

$$f(x) = \begin{cases} -3 & \text{for } -\pi < x < 0 \\ \frac{1}{2}x & \text{for } 0 < x < \pi \\ f(x + 2\pi) & \end{cases}$$

Determine the Fourier series, and hence calculate its percentage second harmonic.

(20 marks)

4. (a) Find the Laplace transform of the function

$$f(t) = e^{-t} \sinh^2 t$$

(4 marks)

(b) Find the inverse Laplace transform of

$$\frac{S + 6S - 7}{(S + 2)(S^2 + 3)}$$

(6 marks)

(c) In the manufacture of a certain alloy two metals react.
If x is the amount of one metal and y the amount of the other, the differential equations describing how the amounts of the two metals vary with time as they react is given by:

$$\frac{dy}{dt} + 3x = e^{-2t}$$

$$\frac{dx}{dt} - 3y = e^{2t}$$

Use Laplace transforms to determine the amount of metal x at any time t given that at $t = 0$,
$x = y = 0$. (10 marks)

5. (a) Solve the following differential equation.

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$$

given that at $x = 0$, $y = 0$ and

$$\frac{dy}{dx} = \frac{-2}{27}$$

(10 marks)

(b) A certain radioactive material is know to decay at a rate proportional to the amount present. If initially there is 250 milligrams of the material present and after two hours it is observed that the material has lost 10 percent of its original mass,

determine:-

(i) an expression for the mass of the material remaining at any time t;
(ii) the time taken for the amount of material to reduce to 125 milligrams;
(iii) the amount remaining after 48 hours. (10 marks)

6. (a) The pressure P and volume V of a gas are related by the equation
$PV^{1.4} = C$
Find the approximate percentage change in C when the pressure is increased by 3 percent and the volume is decreased by 0.9 percentage using partial differentiation. (7 marks)

(b) (i) If $Ø = x^2y + xz^2$ determine grad Ø at the point P (1,3,2). (3 marks)

(ii) Forces of magnitude 5N, 3N, and 1N act in the directions 6i +2j + 3k, 3i - 2j + 6k and 2i - 3j - 6k respectively on a particle which is displaced from the point (2,-1,-3) to (5, -1, 1). Find the work done by the forces. (10 marks)

7. (a) Evaluate

$$\int_0^{\pi/2}\int_0^{x} 2x \sin y \, dy \, dx.$$ (5 marks)

(b) Use double integration to determine the area bounded by the curves

$y = x^2$ and $y = 5x - x^2$ (8 marks)

(c) Find the volume of the solid bounded by the planes $z = 0$, $x = 1$, $x = 3$, $y = -1$, $y = 2$ and the surface

$z = x^2 + y^2 - 4$. (7 marks)

8. (a) Three independent examiners are to review a project. The probabilities that the project will be approved by the three examiners are $\frac{5}{7}$, $\frac{4}{7}$ and $\frac{3}{7}$ respectively. Find the probability that:

(i) all reject the project;
(ii) all approve the project;
(iii) at least two approve the project. (6 marks)

(b) A random variable x has the following probability density function.

$$f(x) = \begin{cases} kx(2-x) & 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the:

(i) value of k;
(ii) mean, mode and median;
(iii) variance of the variable x. (14 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\, t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}\,F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$