2101/301 2106/301
2102/301 2107/301
2103/301 2108/301
2104/301
2105/301
MATHEMATICS
Oct./Nov. 2002
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION)
DIPLOMA IN MECHANICAL ENGINEERING (PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING
DIPLOMA IN AGRICULTURAL ENGINEERING (FARM POWER MACHINERY OPTION)
DIPLOMA IN MECHANICAL ENGINEERING (FABRICATION TECHNOLOGY AND METALLURGY OPTION)
DIPLOMA IN AERONAUTICAL ENGINEERING
DIPLOMA IN MECHANICAL ENGINEERING (MATERIALS TECHNOLOGY AND METALLURGY OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer book
Mathematical tables/calculator.

Answer any **FIVE** of the following **EIGHT** questions.
All answers carry equal marks.
Maximum marks for each part of a question are indicated.
Abridged tables of Laplace transforms and the standard normal distribution are included.

This paper consists of 5 printed pages

Turn over

1. (a) Determine all the fourth roots of $z = 3 + j\,7$ (7 marks)

(b) A particle moves in space so that at time t its position is stated as $x = 2t + 3$, $y = t^2 + 3t$ and $z = t^3 + 2t^2$. Determine the components of its velocity and acceleration in the direction of the vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ when $t = 1$. (8 marks)

(c) Given that $z = x^2 + y^2 + 2xy$

show that $x\frac{\delta z}{\delta x} + y\frac{\delta z}{\delta y} = 2z$ (5 marks)

2. (a) A random variable x follows a normal distribution with mean μ and standard deviation σ. If 12·3% of the distribution is greater than 60 and 5·89% is less than 40, find μ and σ. (8 marks)

(b) A continuous random variable x has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx(1-x), & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the:

(i) value of the constant k

(ii) mean and the standard deviation of x.

(12 marks)

3. (a) Solve the differential equation $x(x^2 + y^2)\frac{dy}{dx} - y^3 = x^3 + 2x^2y$ given that when $x = 1$ $y = 1$. (11 marks)

(b) A tank contains 60 litres of brine containing 3kg of salt. Another solution with 0·5kg of salt per litre pass in at a rate of 5 litres per second and the mixture pass out at the same rate. Determine the amount of salt present in the tank after t seconds. (9 marks)

4. (a) Given that:

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 1 \\ 0 & 4 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 6 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 6 & 4 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \end{pmatrix}$$

Determine:

(i) A + D

(ii) CA

(iii) CD

(iv) B – D.

(8 marks)

(b) Determine the inverse matrix of

$$\begin{pmatrix} 1 & 2 & 2 \\ 3 & -1 & 4 \\ 3 & 2 & -1 \end{pmatrix}$$

Hence solve the following simultaneous equations:

$$x + 2y + 2z = 4$$

$$3x - y + 4z = 25$$

$$3x + 2y - z = -4$$

(12 marks)

5. (a) (i) Express $\cos at$ and $\sin at$ in exponential form using Eurer formula

(ii) Hence determine the lapse transform of $\cos at$ from first principles. (4 marks)

(b) Use laplace transforms to solve the simultaneous equations

$$2\frac{dx}{dt} - 6x + 3y = 0$$

$$3\frac{dy}{dt} - 3y - 2x = 0$$

given that at $t = 0$, $x = 3$ and $y = 1$ (16 marks)

6. (a) Evaluate $\int_{-1}^{1}\int_{1}^{2}(x^2 + y^2)\,dx\,dy$ (4 marks)

(b) A thin plate has variable density $d = x + y + xy$. It is bounded by $y = \frac{1}{2}x$, $x = 4$ and $y = 0$. Determine its centre of gravity (16 marks)

7. (a) The root of the equation $x^3 + 2x^2 - 5x = 1$ is near 1·4. Use Newton Raphson method to find the root correct to three significant figures. (9 marks)

(b) Determine the first three terms of the Maclaurins series of $f(x) = \sec x$

Hence determine $\int_0^2 x^2 \sec x\,dx$. (11 marks)

8. Determine the Fourier series for the following function

$$f(x) = \begin{cases} 1 - x & -\pi < x < 0 \\ 1 + x & 0 < x < \pi \end{cases}.$$ (20 marks)

TABLE OF LAPLACE TRANSFORMS

f(t)	F(s)
0	0
1	$1/s$
k	k/s
t	$1/s^2$
t^n	$n!/s^{n+1}$
$e^{\alpha t}$	$1/(s - \alpha)$
$\sin \omega t$	$\omega/(s^2 + \omega^2)$
$\cos \omega t$	$s/(s^2 + \omega^2)$
$\sin (\omega t + \phi)$	$(s \sin \phi + \omega \cos \phi)/(s^2 + \omega^2)$
$\cos (\omega t + \phi)$	$(s \cos \phi - \omega \sin \phi)/(s^2 + \omega^2)$
$\sinh \beta t$	$\beta/(s^2 - \beta^2)$
$\cosh \beta t$	$s/(s^2 - \beta^2)$
$\frac{1}{a - b}(e^{at} - e^{bt})$	$1/(s - a)(s - b)$
$\frac{1}{a - b}(ae^{at} - be^{bt})$	$s/(s - a)(s - b)$
$1 - e^{\alpha t}$	$-\alpha/s(s - \alpha)$
$1 - \cos \omega t$	$\omega^2/s(s^2 + \omega^2)$
$e^{\alpha t} f(t)$	$F(s - \alpha)$
$e^{\alpha t} t^n$	$n!/(s - \alpha)^{n+1}$
$e^{\alpha t} \sin \omega t$	$\omega/[(s - \alpha)^2 + \omega^2]$

NORMAL DISTRIBUTION FUNCTION

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	9.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998