2207/301
MATHEMATICS
June/July 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING AVIONICS (COMMUNICATION AND NAVIGATION OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non programmable scientific calculator;

Tables of Laplace transforms and the Normal distribution curve.

Answer ***FIVE*** *of the* ***EIGHT*** *questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are as shown.

Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrices:

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 3 & 5 & 0 \\ 0 & -1 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -3 & 3 \\ -2 & 1 & -1 \\ 1 & 1 & 2 \end{pmatrix}$$

Determine $(B^T A)^{-1}$. (10 marks)

(b) Three forces, F_1, F_2 and F_3 in Newtons are acting on a rigid body such that:

$$F_1 + F_2 + 3F_3 = 60$$
$$2F_1 + 3F_2 + 4F_3 = 100$$
$$4F_1 + 2F_2 + 10F_3 = 190$$

Use Crammer's rule to determine the magnitudes of the forces. (10 marks)

2. (a) Given that X_n is an approximation of the root of the equation:

$X^4 - X^2 + 30 = 0$

(i) Show, using Newton-Raphson method that a better approximation X_{n+1} is given by:

$$X_{n+1} = \frac{3X_n^4 - X_n^2 + 30}{4X_n^3 - 2X_n}.$$

(ii) Starting with $X_o = 2$ determine, correct to six decimal places, the root of the equation. (12 marks)

(b) Given that a graph of a function f (x) passes through the points (1, 4.75) and (2, 26.0), use linear interpolation to determine f(1.8). (3 marks)

(c) Calculate the rate at which the volume of a cone is changing when the height is increasing at the rate of 0.5 cm/s and the radius is decreasing at the rate of 0.3 cm/s at the instant when radius is 25 cm and the height is 30 cm. (5 marks)

3. A curve is given by:

$y = 6x^3 - 8x + 1.$

(a) (i) determine the stationary points and their nature;

(ii) At $x = \frac{3}{20}, y = 0$. Sketch the graph. (12 marks)

(b) Determine the x coordinate of the centroid of the area enclosed by the curve, the x - axis and the line $x = -1$. (8 marks)

4. (a) Given the function:

$$f(t)=\begin{cases}0 & 0\le t\le 5\\ 5 & t\ge 5\end{cases}.$$

(i) sketch the graph of the function;

(ii) determine from first principles, the Laplace transform of the function. (8 marks)

(b) Use Laplace transforms to solve the differential equation:

$$\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y=e^{-7t}$$

Given that when $t=0$, $y=1$ and $\frac{dy}{dt}=3$. (12 marks)

5. (a) Given that complex numbers: $z_1=5-j7$ and $z_2=-2+j3$, express:

$\frac{z_1}{z_2}$ in the form $re^{j\theta}$. (6 marks)

(b) Use De-Moivres theorem to prove the identity: $\sin 4\theta=4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta$. (4 marks)

(c) Solve the equation:

$$z^3-5+j\sqrt{2}=0.$$ (10 marks)

6. Solve the differential equations:

(a) $\frac{dy}{dx}-\frac{7y}{x}=x^5$; (6 marks)

(b) $\frac{d^2y}{dt^2}-10\frac{dy}{dt}+25y=3e^{5t}$

Given that when $t=0$, $y=3$ and $\frac{dy}{dt}=1$. (14 marks)

7. (a) A random variable x follows the Poisson distribution with variance 3. Determine the probability that x is at most 2. (5 marks)

(b) The diameters of 1,000 washers are approximately normally distributed with a mean of 172 mm and a standard deviation of 7.2 mm. Two hundred samples each of size 36 are drawn from their population and the means computed. Determine the:

(i) mean and standard error of the sampling distribution of the means.

(i) probability that the sample mean lies between 171 mm and 174 mm.

(iii) value of the constant C such that:

$$P(\bar{x}\le C)=0.013.$$ (15 marks)

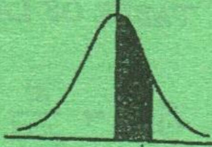
8. A function f (x) is defined by:

$$f(x)=\begin{cases} 3x-x^2 & -\pi < x < \pi \\ f(x+2\pi) & \end{cases}.$$

(a) Sketch the function for $-\pi \leq x \leq 3\pi$. (2 marks)

(b) Find the Fourier series for the function. (18 marks)

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

TABLE OF LAPLACE TRANSFORM FORMULAS

$\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^n}\right] = \dfrac{1}{(n-1)!}t^{n-1}$

$\mathcal{L}[e^{at}] = \dfrac{1}{s-a}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s-a}\right] = e^{at}$

$\mathcal{L}[\sin at] = \dfrac{a}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^2+a^2}\right] = \dfrac{1}{a}\sin at$

$\mathcal{L}[\cos at] = \dfrac{s}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{s}{s^2+a^2}\right] = \cos at$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \dfrac{1}{s}\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$\mathcal{L}[e^{at}f(t)] = F(s-a)$ $\qquad$ $\mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$

Second Differentiation Formula

$\mathcal{L}[t^n f(t)] = (-1)^n \dfrac{d^n}{ds^n}\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$

Second Shift Formula

$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)]$ $\qquad$ $\mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$

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