2506/303
2507/303
ENGINEERING MATHEMATICS III
June/July 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION)
DIPLOMA IN AERONAUTICAL ENGINEERING (AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Drawing instruments;*
- *Mathematical tables/Non-programmable calculator.*

*This paper consists of **EIGHT** questions.*
*Answer **FIVE** questions in the answer booklet provided.*
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 3 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Use the Newton-Raphson method to solve the equation:

$$x^4 - 12x + 7 = 0$$ near 1.9 correct to six decimal places. (11 marks)

(b) Table 1 represents a polynomial $f(x)$.

Table 1

x	1.5	2	2.5	3	3.5	4
$f(x)$	3.375	7	13.625	24	38.875	59

Use the Gregory-Newton forward difference interpolation to determine $f(2.4)$. (9 marks)

2. (a) Determine a matrix whose eigen values are $\lambda_1 = 2$, $\lambda_2 = 1$ and $\lambda_3 = -1$ with corresponding eigen vectors $\underset{\sim}{e}_1 = (1\,3\,1)^T$, $\underset{\sim}{e}_2 = (3\,2\,1)^T$ and $\underset{\sim}{e}_3 = (1\,0\,1)^T$ respectively. (11 marks)

(b) A linear time invariant system is modelled by a vector-differential equation

$$\frac{d\underset{\sim}{x}}{dt} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \underset{\sim}{x}$$

Determine the state transition matrix $\phi(t)$. (9 marks)

3. (a) Expand $f(x) = x$, $0 \leq x \leq \pi$ in half-range sine series. (8 marks)

(b) Determine the Fourier series of $f(t) = \pi + t^2, -\pi \leq t \leq \pi$ (12 marks)

4. (a) Given that $U(x,y) = 4x^2 - 5x - 4y^2$

(i) show that $U(x,y)$ is harmonic;

(ii) determine a conjugate harmonic function $V(x,y)$ such that $f(z) = u + jv$ is analytic.

(12 marks)

(b) The circle $|z| = 2$ is mapped into the w-plane by the transformation:

$$W = \left(\frac{Z + j}{Z - 2j}\right)$$

Determine the:

(i) image of the circle in the W-plane;

(ii) centre and radius of the image circle.

(8 marks)

5. (a) Determine the work done in moving a particle once round the circle

$x^2+y^2=9$, $Z=0$ by the force field

$\underset{\sim}{F}=(2x-y-z)\underset{\sim}{i}+(x+y-z^2)\underset{\sim}{j}+(3x-2y+4z)\underset{\sim}{k}$ (9 marks)

(b) Verify Green's theorem in the plane for $\oint_c (10x^2-8y^2)dx+(5y-6xy)dy$; where c is boundary of the region defined by:

$x=0$, $y=0$ and $x+y=1$. (11 marks)

6. (a) Use a double integral to determine the volume bounded by the xy plane; the paraboloid $2Z=x^2+y^2$ and the cylinder $x^2+y^2=4$. (7 marks)

(b) Find the surface area of the sphere $x^2+y^2+z^2=9$ lying inside the cylinder $x^2+y^2=3y$. (13 marks)

7. (a) Use the divergence theorem to evaluate $\int\int_s \underset{\sim}{F} o d\underset{\sim}{s}$, where $\underset{\sim}{F}=x^3\underset{\sim}{i}+y^3\underset{\sim}{j}+Z^3\underset{\sim}{k}$ and s is the surface of the sphere $x^2+y^2+z^2=9$. (7 marks)

(b) Verify Stokes' theorem for the vector field, $\underset{\sim}{F}=(2x-y)\underset{\sim}{i}-yzj-y^2z\underset{\sim}{k}$ over the upper half of the surface of $x^2+y^2+z^2=1$ bounded by its projection on the xy plane. (13 marks)

8. (a) (i) Determine the half-range Fourier sine series of the function $f(x)=x$, $0\le x\le 2$.

(ii) By setting $x=1$ in (i) show that $\frac{\pi}{4}=\sum_{k=1}^{\alpha}\frac{(-1)^{k+1}}{2k-1}$

(10 marks)

(b) Determine the eigen values and the corresponding eigen vectors of matrix

$A=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$ (10 marks)

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