2207/301
MATHEMATICS
Oct./Nov. 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING AVIONICS (COMMUNICATION AND NAVIGATION OPTION)

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non programmable scientific calculator;
*Answer **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrices:

$$P=\begin{pmatrix}1 & -1 & 2\\ 3 & -2 & 1\\ 1 & 1 & 0\end{pmatrix},\ Q=\begin{pmatrix}-2 & 1 & 4\\ -1 & 2 & 1\\ 3 & 0 & 1\end{pmatrix} \text{ and } R=\begin{pmatrix}5 & 1 & -2\\ 1 & 1 & -1\\ 3 & -1 & 1\end{pmatrix}.$$

Determine:

(i) $P+3Q$;

(ii) $QR-P$. (5 marks)

(b) Use the inverse matrix method to solve the simultaneous:

$$4x+2y+3z=-3$$
$$3x+4y+z=-6$$
$$5x+y-5z=8$$

(15 marks)

2. (a) The position vector of a point P is $P=\begin{pmatrix}2\\5\\1\end{pmatrix}$. Given that $\underset{\sim}{PQ}=\begin{pmatrix}5\\10\\15\end{pmatrix}$ determine the:

(i) position vector q of the point Q;

(ii) coordinates of a point S which divides PQ in the ratio 3:2. (6 marks)

(b) Determine the angle between the two vectors
$\underset{\sim}{a}=3\underset{\sim}{i}-2\underset{\sim}{j}+5\underset{\sim}{k}$ and $\underset{\sim}{b}=2\underset{\sim}{i}+10\underset{\sim}{j}+7\underset{\sim}{k}$.

(8 marks)

(c) Solve for t in the equation $\sinh 2t=2$, correct to 3 decimal places. (6 marks)

3. (a) The average number of accidents occuring weekly on a certain highway is 2. If the number of accidents follow the Poisson distribution, find the probability that in a given week:

(i) no accident occurs;

(ii) exactly two accidents occur;

(iii) at most two accidents occur. (6 marks)

(b) Data in table 1 shows the distribution of masses in grams of a hundred articles produced in a factory.

Mass (g)	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60
Number	5	14	30	28	17	6

Compute the measure of skewness for the distribution. (14 marks)

4. (a) Determine:

(i) The Laplace transform of the function $F(t) = t^2 \cos 2t$.

(ii) The inverse Laplace transform of the function: $F(s) = \dfrac{2s}{s^2 - 8s + 21}$. (10 marks)

(b) Use Laplace transforms to solve the differential equation:

$\dfrac{d^2x}{dt^2} + 3\dfrac{dx}{dt} + 2x = 3\cos 2t$. Given that when t = 0, x = 0 and $\dfrac{dx}{dt} = 4$. (10 marks)

5. (a) A curve is given by the parametric equations $x = 5t$, $y = 5t^2$. Determine the radius of curvature of the curve at the point where t = 1. (9 marks)

(b) Given the function: $z = 3xy - y^3 + (y^2 - 3x)^{\frac{3}{2}}$, show that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$. (6 marks)

(c) Determine the change in volume of a cone when the radius is decreased by 0.05 cm while height is decreased by 0.5 cm given that radius r = 2.5 cm and h = 4 cm. (5 marks)

6. (a) Given that x_n is an approximation to the root of the equation:

$x^3 - 5x^2 + 6 = 0$.

(i) Use Newton-Raphson method to show that a better approximation is given by:

$x_{n+1} = \dfrac{2x_n^3 - 5x_n^2 - 6}{3x_n^2 - 10x_n}$.

(ii) Starting with $x_0 = 2$, determine the root correct to five decimal places. (11 marks)

(b) Use Simpson's rule with six strips to evaluate:

$\int_0^6 \dfrac{dx}{1 + x^3}$. (9 marks)

7. Solve the differential equations:

(a) $\dfrac{dx}{dt} + x = 9$ given that when t = 0, x = 5. (7 marks)

(b) $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} + x = 7e^t$.

Given that when t = 0, x = 3 and $\dfrac{dx}{dt} = 1$. (13 marks)

8. (a) A function is derived by:

$$f(x)=\begin{cases} x^2 & -5\le x\le 5 \\ f(x+10), & \end{cases}.$$

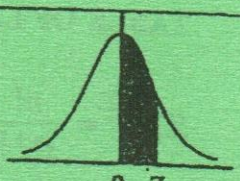
(i) sketch the function for $-5\le x\le 10$; (2 marks)

(ii) determine the Fourier series for the function. (12 marks)

(b) Expand the function $f(x)=2\left(1-\frac{x}{5}\right)$ in a half range sine series for $0<x<5$.

(6 marks)

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000