2101/301 2105/301
2102/301 2106/301
2103/301 2107/301
2104/301 2108/301
MATHEMATICS
Oct./Nov. 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

# DIPLOMA IN MECHANICAL ENGINEERING
# (PRODUCTION OPTION)
# (PLANT OPTION)
# (FABRICATION TECHNOLOGY AND METALLURGY OPTION)
# (MATERIALS TECHNOLOGY AND METALLURGY OPTION)
# DIPLOMA IN AUTOMOTIVE ENGINEERING
# DIPLOMA IN CONSTRUCTION PLANT ENGINEERING
# DIPLOMA IN AGRICULTURAL ENGINEERING
# (FARM POWER AND MACHINERY OPTION)
# DIPLOMA IN AERONAUTICAL ENGINEERING
# (AIRFRAMES AND ENGINES OPTION)

MATHEMATICS

**3 hours**

## INSTRUCTIONS TO CANDIDATES

*You should have the following for this examination:*
*Answer booklet*
*Mathematical tables/Non-programmable scientific calculator;*
*Drawing instruments.*
*This paper consists of **EIGHT** questions.*
*Answer **FIVE** questions in the answer booklet provided.*
*All questions carry equal marks.*
*Maximum marks for each part of a question are as shown.*
***Candidates should answer the questions in English.***

**This paper consists of 6 printed pages.**

**Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**



**Turn over**

1. (a) Solve the differential equation:

$$x\frac{dy}{dx} - y = \frac{x}{x-1}$$ (6 marks)

(b) Use the method of undetermined coefficients to solve the differential equation:

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 8x^2$, given that when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 3$. (14 marks)

2. (a) Given the matrices

$$A = \begin{bmatrix} 4 & -3 & 7 \\ -2 & -5 & 1 \\ -1 & 6 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 5 \\ -3 & -4 & -2 \end{bmatrix}$$

Determine:

(i) $AB$;

(ii) $N = A + B$;

(iii) $N^{-1}$.

(12 marks)

(b) Use Cramer's rule to solve the following system of equations:

$2x + 3y + z = 1$

$x - y + z = 4$

$5x + y + 3z = 10$

(8 marks)

3. (a) Determine the Laplace transform of $f(t) = \sinh 2t$ from first principles. (7 marks)

(b) A machine component moves in such a way that its displacement $x(t)$ satisfies the differential equation:

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 6.$$

Given that when $t = 0$, $x = 2$ and $\frac{dx}{dt} = 0$, use Laplace transforms to solve the differential equation.

(13 marks)

4. Given the periodic function:

$$f(t)=\begin{Bmatrix} 2+t & ; & -2<t<0 \\ 2-t & ; & 0<t<2 \\ f(t+4) & & \end{Bmatrix}$$

(a) sketch the graph for $-6<t<6$; (3 marks)

(b) determine its Fourier series; (14 marks)

(c) by letting $t=0$ in the series obtained in (b), show that:

$$\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}.$$ (3 marks)

5. (a) (i) If $X_n$ is an approximation to the cube root of 17, use the Newton-Raphson method to show that a better approximation to the cube root of 17 if given by:

$$X_{n+1}=\frac{2X_n^3+17}{3X_n^2}.$$

(ii) Hence determine the cube root of 17 correct to five decimal places, taking $x_0=3$.

(8 marks)

(b) The data in table 1 was obtained from an experiment.

**Table 1**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | –24 | –17 | 14 | 81 | 196 | 371 | 618 | 949 |

Use the Newton-Gregory finite difference interpolation formulae to determine:

(i) $f(1.4)$;

(ii) $f(8.5)$.

(12 marks)

16. (a) The heights of students in a certain institution are normally distributed with a mean of 160 cm and a standard deviation of 10 cm. Determine the:

(i) probability that a student chosen at random will have a height greater than 175 cm;

(ii) proportion of students with heights between 143 cm and 168 cm;

(iii) number of students likely to have a height less than 152 m, if the total number of students in the institution is 1000.

(11 marks)

(b) A continuous random variable X is modelled by a probability density function.

$$f(x)=\begin{Bmatrix} kx; & 0<x<2 \\ k\ ; & 2<x<x5 \\ 0\ ; & elsewhere \end{Bmatrix}$$

where $k$ is a constant.

Determine the:

(i) value of constant $k$;

(ii) mean value of $x$;

(iii) median. (9 marks)

17. (a) Use vector analysis to determine the area of the triangle with vertices at points P(1, 0, 1), Q(–2, 1, 3) and R(4, 2, 5). (7 marks)

(b) Given the vector $\underset{\sim}{A}=x^2yz\underset{\sim}{i}+xy^3\underset{\sim}{j}+xz^2\underset{\sim}{k}$, determine $\nabla\cdot(\nabla\times\underset{\sim}{A})$. (5 marks)

(c) Determine the stationary points of the function $f(x,y)=y^3+3x^2y-6x^2-6y^2+2$. (8 marks)

8. (a) Evaluate the following integrals:

(i) $\int_0^1\int_0^{2y}\{4x\sqrt{y}+y\}\,dxdy$;

(ii) $\int_0^{\frac{\pi}{2}}\int_0^{y}\int_0^{x}\cos(x+y+z)\,dzdxdy$. (12 marks)

(b) Use triple integral to determine the volume of the solid bounded by the surfaces:

$z=16-3x^2-y^2$ and $z=x^2+3y^2$. (8 marks)

## TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

### First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

### First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

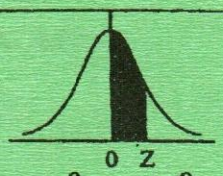
### Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

### Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Areas under the Standard Normal curve from 0 to Z




| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0754 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2258 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2518 | 0.2549 |
| 0.7 | 0.2580 | 0.2612 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2996 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |

**THIS IS THE LAST PRINTED PAGE.**