

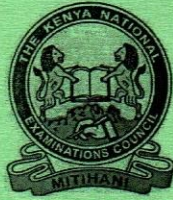
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ENGINEERING MATHEMATICS III

Oct./Nov. 2018

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)**

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

The candidate should have the following for this examination:

Answer booklet;

Mathematical table/Scientific calculator;

Drawing instruments.

*Answer **FIVE** of the **EIGHT** questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are as shown.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

- 1/ (a) (i) Given that x_n is an approximation to the root of the equation $x^3 + 6x - 5 = 0$, use the Newton-Raphson method to show that a better approximation is given by:

$$x_{n+1} = \frac{2x^3 + 5}{3x^2 + 6}.$$

- (ii) Taking $x_0 = 0.5$, determine the root, correct to four decimal places.

(9 marks)

- (b) A polynomial function $f(x)$ is represented by the data in **Table 1**.

Table 1

x	-1	0	1	2	3	4
$f(x)$	-2	-1	2	19	62	143

Use the Newton-Gregory forward difference interpolation formula to determine $f(x)$, and find:

(i) $f(-3.2)$;

(ii) $f(5)$.

(11 marks)

- 2/ (a) Determine the eigenvalues and corresponding eigenvectors of the matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

(12 marks)

- (b) A dynamic system is modelled by the vector matrix differential equation:

$$\frac{d\underline{x}}{dt} = A\underline{x}, \text{ where } A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \text{ and } \underline{x}(t) \text{ is the system state vector.}$$

Determine the system state transition matrix, $\Phi(t)$.

(8 marks)

- 3/ (a) Given the function $f(z) = z^2 - 2z + 3$, where the complex variable $z = x + jy$,

(i) express $f(z)$ in the form $u(x, y) + jv(x, y)$;

(ii) show that u and v are harmonic functions.

(8 marks)

- (b) The circle $|z| = 2$ in the z -plane is mapped onto the w -plane by the transformation

$$w = \frac{z + 3j}{z + j}. \text{ Determine the centre and radius of the image circle.}$$

(12 marks)

4. (a) Find the half-range Fourier cosine series of the function $f(t) = \pi^2 + t^2, 0 < t < \pi$.
(8 marks)

(b) A function $f(t) = \begin{cases} \pi, & -\pi < t < 0 \\ \pi - t, & 0 < t < \pi \\ f(t+2\pi) \end{cases}$

Sketch the graph of $f(t)$ in the interval $-\pi < t < 3\pi$, and determine its Fourier series representation.
(12 marks)

5. (a) Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{y}{\sqrt{x^2+y^2}} dy dx$$

(5 marks)

(b) Change the order of integration, and hence determine the value of the integral:

$$\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3) dy dx.$$

(8 marks)

(c) Use a triple integral to find the volume of the solid in the first octant bounded by the coordinate planes, and the plane $x + y + 2z = 1$.
(7 marks)

6. Verify Green's theorem in the plane for the line integral $\oint_T xy dx + (x + y^2) dy$, where T is the perimeter of the triangle with vertices (0,0), (1,1) and (-1,1) oriented counterclockwise.
(20 marks)

7. (a) The eigenvalues of a 2×2 matrix A are $\lambda_1 = 3$ and $\lambda_2 = 2$ with corresponding eigenvectors $e_1 = [-2 \ 1]^T$ and $e_2 = [-1 \ 1]^T$.

Determine:

(i) the modal matrix M and spectral matrix Λ of A;

(ii) the matrix A;

(iii) A^2 .

(10 marks)

(b) (i) Determine the half-range Fourier sine series of $f(t) = t, 0 < t < \pi$.

(ii) Use the result in (i) to show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

(10 marks)

8. (a) Evaluate $\iint_S \underline{F} \cdot d\underline{S}$, where the vector field $\underline{F} = (y-x)\underline{i} + (y-z)\underline{j} + (x-z)\underline{k}$ and S is the part of the plane $x + y + z = 1$ in the first octant. (11 marks)
- (b) Use the divergence theorem to evaluate

$\iint_S \underline{F} \cdot d\underline{S}$ for the vector field $\underline{F} = -y\underline{i} - yz\underline{j} + z^2\underline{k}$, where S is the upper hemisphere $x^2 + y^2 + z^2 = 4, z \geq 0$, oriented by outward unit normals. (9 marks)

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