2506/303
2507/303
ENGINEERING MATHEMATICS III
Oct./Nov. 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

The candidate should have the following for this examination:
Answer booklet;
Mathematical table/Scientific calculator;
Drawing instruments.
*Answer **FIVE** of the **EIGHT** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) (i) Given that x_n is an approximation to the root of the equation $x^3 + 6x - 5 = 0$, use the Newton-Raphson method to show that a better approximation is given by:

$$x_{n+1} = \frac{2x^3 + 5}{3x^2 + 6}.$$

(ii) Taking $x_0 = 0.5$, determine the root, correct to four decimal places.

(9 marks)

(b) A polynomial function $f(x)$ is represented by the data in **Table 1**.

Table 1

x	-1	0	1	2	3	4
$f(x)$	-2	-1	2	19	62	143

Use the Newton-Gregory forward difference interpolation formula to determine $f(x)$, and find:

(i) $f(-3.2)$;

(ii) $f(5)$.

(11 marks)

2. (a) Determine the eigenvalues and corresponding eigenvectors of the matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

(12 marks)

(b) A dynamic system is modelled by the vector matrix differential equation:

$\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}$, where $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ and $\underset{\sim}{x}(t)$ is the system state vector.

Determine the system state transition matrix, $\Phi(t)$.

(8 marks)

3. (a) Given the function $f(z) = z^2 - 2z + 3$, where the complex variable $z = x + jy$,

(i) express $f(z)$ in the form $u(x,y) + jv(x,y)$;

(ii) show that u and v are harmonic functions.

(8 marks)

(b) The circle $|z| = 2$ in the z-plane is mapped onto the w-plane by the transformation $w = \frac{z+3j}{z+j}$. Determine the centre and radius of the image circle.

(12 marks)

4. (a) Find the half-range Fourier cosine series of the function $f(t)=\pi^2+t^2, 0<t<\pi$.

(8 marks)

(b) A function $f(t)=\begin{cases} \pi, & -\pi<t<0 \\ \pi-t, & 0<t<\pi \\ f(t+2\pi) & \end{cases}$

Sketch the graph of $f(t)$ in the interval $-\pi<t<3\pi$, and determine its Fourier series representation.

(12 marks)

5. (a) Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{y}{\sqrt{x^2+y^2}} dydx$$

(5 marks)

(b) Change the order of integration, and hence determine the value of the integral:

$$\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3) dydx .$$

(8 marks)

(c) Use a triple integral to find the volume of the solid in the first octant bounded by the coordinate planes, and the plane $x+y+2z=1$.

(7 marks)

6. Verify Green's theorem in the plane for the line integral $\oint_T xydx+(x+y^2)dy$, where T is the perimeter of the triangle with vertices (0,0), (1,1) and $(-1,1)$ oriented counterclockwise. 10

(20 marks)

7. (a) The eigenvalues of a 2×2 matrix A are $\lambda_1=3$ and $\lambda_2=2$ with corresponding eigenvectors $e_1=[-2\ \ 1]^T$ and $e_2=[-1\ \ 1]^T$.

Determine:

(i) the modal matrix M and spectral matrix $\wedge$ of A; 5

(ii) the matrix A;

(iii) A^2.

(10 marks)

(b) (i) Determine the half-range Fourier sine series of $f(t)=t, 0<t<\pi$.

(ii) Use the result in (i) to show that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+-\quad$.

(10 marks)

8. (a) Evaluate $\iint_S \underset{\sim}{F} \cdot d\underset{\sim}{S}$, where the vector field $\underset{\sim}{F} = (y-x)\underset{\sim}{i} + (y-z)\underset{\sim}{j} + (x-z)\underset{\sim}{k}$ and S is the part of the plane $x+y+z=1$ in the first octant. (11 marks)

(b) Use the divergence theorem to evaluate

$\iint_S \underset{\sim}{F} \cdot d\underset{\sim}{S}$ for the vector field $\underset{\sim}{F} = -y\underset{\sim}{i} - yz\underset{\sim}{j} + z^2\underset{\sim}{k}$, where S is the upper hemisphere $x^2+y^2+z^2=4, z \geq 0$, oriented by outward unit normals. (9 marks)

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