2506/203
2507/203
ENGINEERING MATHEMATICS II
Oct./Nov. 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN AERONAUTICAL ENGINEERING (AIRFRAMES AND ENGINES OPTION) (AVIONICS OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;
Drawing instruments;
Mathematical tables/Non-programmable scientific calculator.

*This paper consists of **EIGHT** questions.*
*Answer **FIVE** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) (i) Determine the Maclaurin's series expansion of $f(x) = \ln(2x+1)$ up to the term in x^4.

(ii) Hence evaluate $\int_0^1 \frac{\ln(2x+1)dx}{\sqrt{x}}$ giving the answer correct to 3 decimal places.

(11 marks)

(b) (i) Determine the Taylor's series expansion for the function $\sin(a+h)$ in ascending powers of h up to the term in h^3.

(ii) Hence determine the value of $\sin(30.5°)$ giving the answer correct to **five** decimal places.

(9 marks)

2. (a) Solve the differential equation $2xy\frac{dy}{dx} = x^2 + y^2$. (5 marks)

(b) Use the method of undetermined coefficients to solve the following differential equation, $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 4\cos 3x$, given that when $x = 0, y = 3$ and $\frac{dy}{dx} = 0$.

(15 marks)

3. (a) Determine the value of p such that the three vectors.

$\underset{\sim}{A} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$; $\underset{\sim}{B} = \underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{C} = 3\underset{\sim}{i} + P\underset{\sim}{j} + 5\underset{\sim}{k}$ are coplanar.

(4 marks)

(b) Given the vectors $\underset{\sim}{A} = 2x^2y\underset{\sim}{i} + (xy^2 + yz)\underset{\sim}{j} + xz^2\underset{\sim}{k}$ and $\underset{\sim}{B} = xyz^2\underset{\sim}{i} + 3y^2z^2\underset{\sim}{j} + xy^2z\underset{\sim}{k}$, determine at point (1, -1, 3):

(i) $\nabla \cdot \underset{\sim}{A}$;

(ii) $\nabla \times \underset{\sim}{B}$.

(10 marks)

(c) Given the scalar field $\phi(x,y,z) = x^3y + y + z^2$, determine the directional derivative of ϕ in the direction of vector $\underset{\sim}{A} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ at the point (1, 2, 2). (6 marks)

4. (a) Given the matrices:

$$A = \begin{bmatrix} 3 & -2 & -3 \\ 1 & -2 & 0 \\ 3 & -1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 1 \\ -2 & -3 & 1 \end{bmatrix}$$

Determine:

(i) $N = AB + B$;

(ii) N^{-1}

(12 marks)

(b) Use the result in (a) to solve the following linear simultaneous equations:

$$10x - 3y - 17z = 28;$$
$$x - 4y - 4z = 6;$$
$$3x - 8y - 10z = 16.$$

(6 marks)

(c) Given that $\begin{bmatrix} 2-a & -3 \\ -5 & 6+b \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -5 & 16 \end{bmatrix}$. Determine the values of a and b.

(2 marks)

5. (a) Given that $Z = (x,y) = \dfrac{x-y}{x+y}$, determine the value of $\dfrac{\partial^2 z}{\partial x^2}$ at the point (2,1).

(6 marks)

(b) Determine and classify the stationary points of the function $z = x^3 + y^3 + 4xy$.

(14 marks)

6. (a) From first principles, show that the Laplace transform of $f(t) = \sinh 3t$ is $\dfrac{3}{s^2 - 9}$.

(7 marks)

(b) A machine member moves in such a way that its displacement, $x(t)$ satisfies the differential equation:

$\dfrac{d^2x}{dt^2} - 4\dfrac{dx}{dt} + 4x = 3e^{2t}$; given that when $t = 0$, $x = 2$ and $\dfrac{dx}{dt} = 1$. Use Laplace transforms to determine the expression for displacement. (13 marks)

7. (a) The service life of a fluorescent tube used for lighting a certain factory is normally distributed with a mean of 640 hrs and a standard deviation of 50 hrs. Determine the:

(i) probability that a fluorescent tube chosen at random from the same batch will last between 600 and 690 hrs.

(ii) proportion of tubes that will last less than 620 hrs.

(iii) number of fluorescent tubes likely to last more than 750 hrs, if the number supplied by the factory to a certain store is 1000.

(14 marks)

(b) During working hours, an office switchboard receives telephone calls at random at an average rate of one call every 40 seconds. Giving the answers correct to four decimal places, determine the probability that during a given one minute period:

(i) no call is received;
(ii) at least two calls are received.

(6 marks)

8. (a) Table 1 shows the distribution of lifetimes of a sample of batteries.

Table 1

Lifetimes (hours)	700 - 709	710 - 719	720 - 729	730 - 739	740 - 749	750 - 759	760 - 769
Frequency	3	7	15	38	41	35	21

Determine the:

(i) median;
(ii) mode.

(6 marks)

(b) In an experiment, the temperature of a metal rod was raised from 300 K. The extensions E mm of the rod at selected temperatures T kelvin are shown in Table 2.

Table 2

temp (Kelvin)	300	350	400	450	500	550	600	650	700
Extensions (mm)	0	0.38	0.8	1.22	1.6	2	2.42	2.8	3.18

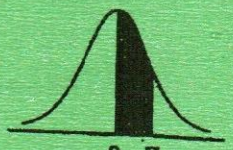
(i) Calculate the product-moment correlation coefficient between the variables.

(ii) Comment on the result of b(i).

(iii) Use the method of least squares to determine the equation of the regression line of extension (E) on temperature (T).

(14 marks)

Areas under the Standard Normal curve from 0 to Z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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