

2506/103

2507/103

ENGINEERING MATHEMATICS I AND
ENGINEERING SCIENCE I

June/July 2017

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL
DIPLOMA IN AERONAUTICAL ENGINEERING
(AIRFRAMES AND ENGINES OPTION)
(AVIONICS OPTION)

MODULE I

ENGINEERING MATHEMATICS I AND ENGINEERING SCIENCE I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Drawing instruments;

Non programmable scientific calculator/Mathematical table;

Answer booklet.

This paper consists of TWO sections; A and B.

Answer THREE questions from Section A and TWO questions from section B in the answer booklet provided.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

SECTION A: ENGINEERING MATHEMATICS I

Answer **THREE** questions from this section.

- ✓ 1. (a) Evaluate the expressions:
- (i) $2 \log_2 8 + \log_2 128 - 5 \log_2 4$
- (ii) $\frac{\log_3 64}{\log_5 81}$ (5 marks)
- (b) (i) Prove that:
- $$2 \sinh^2 \theta = \cosh 2\theta - 1$$
- (ii) If $5e^x - 10e^{-x} = A \cosh x + B \sinh x$, determine the values of A and B. (8 marks)
- (c) Prove the following trigonometric identities:
- (i) $\frac{\cos \theta - \sin \theta}{\cos 2\theta} = \frac{\sec \theta}{1 + \tan \theta}$
- (ii) $\sin(\theta + \phi) + \sin(\theta - \phi) = 2 \sin \theta \cos \phi$ (7 marks)
2. (a) (i) Express $Z = 5 \angle 60^\circ$ in cartesian form.
- (ii) Given $Z_1 = 3j$ and $Z_2 = 3 + j$, evaluate;
- $$\frac{Z_2}{Z_1 + Z_2}$$
- (iii) Represent the complex number $12 - 7j$ on an Argand diagram. (8 marks)
- (b) Solve the following equations:
- (i) $4x^2 - 5x - 6 = 0$ by completing the square method.
- (ii) $6x^2 - x - 1 = 0$ by factorization. (7 marks)

(c) Find the:

(i) polar equation of the curve;

$$x^2 + y^2 = 8y$$

(ii) cartesian equation of the curve represented by the polar equation;

$$r^2 = 9 + 4r \cos \theta$$

(5 marks)

3. (a) (i) Determine the number of five letter words that can be formed using the letters from the word ENGINEERING.

(ii) A section of a department needs three supervisors who are to be chosen from fifteen members. Calculate the number of different ways that this can be done.

(7 marks)

(b) Simplify the expression:

(i) $1 - \cos^2 15^\circ$

(ii) $\cos 150^\circ \sin 60^\circ - \cos 60^\circ \sin 150^\circ$

(iii) $\cos 270^\circ \cos 45^\circ - \sin 270^\circ \sin 45^\circ$

(iv) $1 - 2 \cos^2 41^\circ$

(4 marks)

(c) Solve the equation:

$$2 \cos^2 \theta - \sin \theta - 1 = 0$$

(9 marks)

4. (a) Calculate the volume and the surface area of the following solids:

(i) a hemisphere of radius 7.2 cm.

(ii) a cone of radius 15.2 cm and height 12.0 cm.

(8 marks)

(b) The velocity V of an aeroplane varies with time t according to the equation $V = 60 + 2t^2$ immediately after take off.

- (i) Make a table of V against t for integer values of t between $t = 0$ and $t = 6$.
- (ii) Using the table in (i), determine the area under the $V - t$ graph by Simpson's rule.

(5 marks)

(c) Using basic hyperbolic definitions, evaluate:

(i) $\sinh \frac{\sqrt{3}}{2}$

(ii) $\coth 0.5$

(4 marks)

(d) Solve the equation:

$$8^{4x-1} = 16^{x+1}$$

(3 marks)

5. (a) Given $y = \frac{\ln x}{x^2}$, find $\frac{dy}{dx}$.

(5 marks)

(b) Use implicit differentiation to determine the gradient of the tangent to the curve $x^2 + 3xy + y^2 = x + y + 8$ at the point $(1, -3)$.

(5 marks)

(c) Evaluate the integrals:

(i) $\int x \sin x \, dx$

(ii) $\int_2^3 \frac{13x + 19}{(x-1)(x+3)} \, dx$

(10 marks)

SECTION B: ENGINEERING SCIENCE I

Answer **TWO** questions from this section.

6. (a) Define the following as used in linear motion:

- (i) speed;
- (ii) linear deceleration.

(2 marks)

(b) A body is projected vertically upwards from the ground with an initial speed of 40 m/s. Determine:

- (i) time taken to reach the highest point above the ground;
- (ii) total distance travelled when it strikes the ground;
- (iii) speed at the height of 35 m.

(8 marks)

(c) With reference to simple harmonic motion define the terms:

- (i) displacement;
- (ii) frequency.

(2 marks)

(d) A body of mass 0.5 kg oscillates with simple harmonic motion. If its amplitude and periodic time are 10 cm and 0.2 seconds respectively, calculate the:

- (i) maximum acceleration of the body;
- (ii) maximum force acting on the body;
- (iii) maximum velocity of the body;
- (iv) velocity of the body at a point 4 cm from the central position.

(8 marks)

7. (a) State:

- (i) Charle's law;
- (ii) Pressure law.

(2 marks)

(b) A quantity of gas is expanded in a chamber to three times its original volume. The initial pressure is 400 kPa and the final pressure is 100 kPa. If the initial temperature is 300 °C, determine the final temperature in ° celcius.

(5 marks)

(c) Describe the following thermal processes:

- (i) isothermal;
- (ii) isochoric;
- (iii) isobaric;
- (iv) adiabatic.

(8 marks)

(d) Distinguish between the two modes of heat conduction in metals.

(5 marks)

8. (a) (i) Define density.
- (ii) An alloy is made of two metals x and y of densities 7.8 and 8.9 in g/cm^3 respectively. If the ratio of the metals by mass is $x:y = 7:3$, and assuming that the individual volumes of the metals are not affected, determine the density of the alloy.
- (5 marks)
- (b) (i) Define pressure.
- (ii) The barometer reading in a aeroplane at sea level is 760 mmHg and is 740 mmHg at a point P. Given that the densities of mercury and water are 13.6 g/cm^3 and 1.25 kg/m^3 respectively, determine the altitude at point P.
- Take $g = 10 \text{ m/s}^2$.*
- (5 marks)
- (c) Define:
- (i) heat capacity of a body;
- (ii) specific heat capacity of a substance;
- (iii) specific latent heat of fusion of a substance.
- (3 marks)
- (d) A piece of copper 400 g at 200°C is dipped in a copper can of heat capacity 100 J/K containing 1.0 kg of water at 20°C . Given that the specific heat capacities of water and copper are 4200 J/kg K and 400 J/kg K respectively, determine the final stable temperature, assuming negligible heat loss.
- (7 marks)

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